

SYMMETRY RESTORATION BY
PROJECTION TECHNIQUES
IN NUCLEAR MODEL
CALCULATIONS

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Title and subtitle SYMMETRY RESTORATION BY PROJECTION TECHNIQUES IN NUCLEAR MODEL CALCULATIONS (contains an introduction to and a summary of:) I. The single-nucleon formfactor for knock-out and pick-up reactions in deformed nuclei, II. Investigations of the cranking model wave function with respect to angular momentum, III. Angular momentum projected wave functions, IV. Pairing properties at high rotational frequencies in the cranked HFB model, V. On the interaction between the ground- and s-bands in the CHFB model.			
Abstract Exact projection in order to restore obvious symmetries that have been violated to gain simplicity in nuclear model calculations. Projection of angular momentum, isospin and particle number is performed. In paper I a formalism is developed for calculation of single-nucleon formfactors between intrinsically deformed nuclear states. The importance of using rotationally invariant states in the theoretical analyses is emphasized. The method is applied to the (p,2p)-reaction and results in agreement with experimental data has been obtained for the cases $^{12}\text{C}(p,2p)^{11}\text{B}$ and $^{16}\text{O}(p,2p)^{15}\text{N}$. In paper II various aspects of the cranking model wave function are investigated. It is found that the signature quantum number does not impose any strict selection rule of the angular momenta contained in the wave function. A method for projection of angular momentum from cranking wave functions is suggested and various symmetry aspects is discussed. In paper III it is illustrated how projection of angular momentum affects the density distribution of deformed many-particle wave functions. Paper IV contains a thorough analysis of the cranked HFB model wave function. Noninteracting quasiparticle configurations within the $i_{3/2}$ shell are established and analyzed. The influence of an increasing rotational frequency on the pairing properties of the non-interacting configurations is investigated. Particle number projection increases the self-consistent pairing-gap and an explanation for this is presented. Characteristic labels for all configurations can be found by inspection of the projected wave functions. Paper V exploits the cranked HFB model (CHFB) together with particle number projection in order to analyze the interaction between the two lowest quasiparticle configurations of the CHFB-model. The changes of sign of the interaction as a function of the chemical potential can be explained from certain phase relations between the different particle number components.			
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by

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Civ. ing., Klm.

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Lund 1980



1. Synopsis

This thesis gives a short introduction to and an outline of a dissertation based on the following papers:

- I. H.-B. Håkansson, T. Berggren and R. Bengtsson: The single nucleon formfactor for knock-out and pick-up reactions in deformed nuclei, Nucl. Phys. A306 (1978) 406.
- II. R. Bengtsson and H.-B. Håkansson: Investigation of the cranking model wave function with respect to angular momentum, Z. Physik A288 (1978) 193.
- III. R. Bengtsson and H.-B. Håkansson: Angular momentum projected wave functions, Selected Topics in Nuclear Structure - Proceedings of XVI Winter School, Bielsko-Biala, Poland, 1978, p. 704.
- IV. R. Bengtsson and H.-B. Håkansson: Pairing properties at high rotational frequencies in the cranked HFB model, submitted to Nuclear Physics.
- V. H.-B. Håkansson: On the interaction between the ground- and s-bands in the CHFB model, submitted to Physics Letters.

These papers have a common feature, viz. exact projection in order to restore obvious symmetries that have been violated to gain simplicity in nuclear model calculations.

Paper I presents a method for calculation of single nucleon form factors, defined as overlap integrals between intrinsically deformed nuclear states. Projection of angular momentum and isospin from Nilsson model states is performed and the importance of using rotationally invariant states is demonstrated. Comparison with experimental results is made for the reactions $^{12}\text{C}(p, 2p) ^{11}\text{B}$ and $^{16}\text{O}(p, 2p) ^{15}\text{N}$.

Paper II suggests a method for angular momentum projection from cranking model wave functions and gives a discussion of symmetries in this context.

Paper III reviews some of the results in paper I and II from a somewhat different point of view and demonstrates the effect of angular momentum projection on deformed many-particle wave functions.

Paper IV exploits the $i_{13/2}$ cranking model for investigations of pairing properties at high angular momenta before as well as after particle number projection. By elimination of the interaction between certain quasi-particle levels near the Fermi surface it has been possible to follow various unperturbed configurations up to very high rotational frequencies. The size of the pairing gap as well as the influence of number projection on the pairing gap is investigated.

Paper V uses the model of paper IV in order to gain further insight into the nature of the interaction matrix

element between the zero quasiparticle and the first excited two quasiparticle configurations before and after number projection. The importance of defining the unperturbed configurations in an appropriate way is emphasized. All terms of the Hamiltonian cooperate in producing the characteristic behaviour of the interaction. The clue to a more fundamental explanation of this behaviour may be found in the phase relationships of the different particle number components of the wave functions.

2. Symmetries

In theoretical investigations of the atomic nucleus two essentially different problems are encountered. Firstly, there exists no complete knowledge of the strong forces acting between the individual nucleons and that are responsible for the stability of the nucleus. In atomic and molecular physics, where the basic interaction is known, this difficulty does not arise. Secondly, even if the forces were known it would still not be possible to present an exact solution of the many-body problem when the number of particles exceeds two. Consequently, the scientist of today have had to face the fact that a complete theoretical description of the atomic nucleus is still beyond reach. Nevertheless, fifty years of experimental investigations and theoretical model studies have furnished a very comprehensive description of the nuclear constitution. To a great extent it is the consideration of various invariance principles that has rendered this

progress possible.

The true nuclear Hamiltonian has certain obvious invariances such as for example translational and rotational invariance (see e.g. ref. [1]). As this Hamiltonian is not available, an approximate one, $\hat{H}$, has to be used instead. It is desirable, and often required, that this approximate Hamiltonian obey the same symmetry rules as the exact one. In other words, there are certain symmetry operators $\hat{s}_i$ with which $\hat{H}$ must commute, and the associated dynamical variables are then constants of motion. If it had been possible to solve the eigenvalue problem $\hat{H}\psi = E\psi$ exactly, then also the solutions ψ would follow the symmetry rules imposed. However, there is no way around the above described second fundamental difficulty, i.e. one cannot reach the exact eigenfunction ψ . In order to bypass this dilemma a variety of approximate methods have appeared during the years. Most of them are in one way or another connected with the well-known Hartree Fock (HF) approximation [2].

These approximate methods provide wave-functions which are not eigenfunctions of the original Hamiltonian, $\hat{H}$, and consequently will not possess all the symmetries of ψ . For example, $\hat{H}$ commutes with the operators that generates rotations and hence ψ will have well-defined angular momentum which is generally not the case for the solutions of the HF-equations. In deformed HF, as well as in the Nilsson model [3], the total wave function is constructed as an antisymmetric product, i.e. a single Slater determinant, of deformed single-particle wave functions. This procedure in general leads to wave functions

which do not satisfy rotational invariance.

Different ways to master these difficulties have been suggested. The present thesis is concerned with one of them, namely the method of projection onto states having the desired symmetry. Almost any lost symmetry can be projected out and here the restoration of angular momentum, isospin and particle number is considered.

3. Projection of angular momentum and isospin

Projection of angular momentum has become a valuable link between the HF solution and the physical state it is intended to describe. The first outlines of a projection procedure came in 1953 when Hill and Wheeler [4] formulated a generator coordinate method for collective vibrations. In 1957 Peierls and Yoccoz [5], working along similar lines, developed a useful angular momentum projection procedure, henceforth called the PY method. They introduced the projection operator

$$P_{MK}^J = \frac{2J+1}{8\pi^2} \int d\Omega D_{MK}^{J*}(\Omega) \hat{R}(\Omega),$$

where $\hat{R}(\Omega)$ is the operator for a rotation through the Euler angles $\Omega \equiv (\alpha, \beta, \gamma)$ and $D_{MK}^J(\Omega)$ is its matrix element with respect to a suitably defined basis of angular momentum eigenfunctions. They were then able to show that this operator when applied to an arbitrary trial wave function in a variational approximation scheme yields a state with lower expectation value of the (invariant)

Hamiltonian. Hence the projection leads to an improved wave function which, in addition, has the well-defined angular momentum prescribed by the choice $D_{MK}^J(\Omega)$. The operator P_{MK}^J is the one that has been employed in papers I-III on Nilsson model wave functions. These functions are used here as trial wave functions lacking rotational symmetry but incorporating important long-range correlations. In view of our remark earlier concerning the properties of $\hat{H}$, it may seem strange to project angular momentum in connection with the Nilsson model which does not have a rotationally invariant Hamiltonian. Here the primary interest is focused on the wave functions, with a correct spin as a necessary requirement while the prediction of energies is of secondary importance. In paper I, though, a two-body force was subsequently employed in order to achieve a fair description of the lowest states in ^{11}B .

It is well-known that for a heavy nucleus the exact solution of the PY-integral above is a problem of immense complexity. Therefore, in a large number of papers over the past fifteen years various approximation methods have been proposed and exploited [6]. They generally suffer from the shortcoming of not providing the projected wave functions in an explicit form, which is necessary for the present investigation. Papers I and II describe an exact and purely algebraic method for the solution of the PY-integral. The wave functions are represented in terms of uncoupled Slater determinants, before as well as after the projection. This very convenient representation, the m-scheme, has lately become appreciated also in shell-model

calculations [7] due to its suitability for computer handling. The projection method requires computer time and memory size roughly in proportion to the factorial of the number of particles involved, so it is not applicable to systems with many particles in large configuration spaces.

Angular momentum projection may affect both the wave functions and the energies considerably. The density distributions shown in paper III clearly demonstrate how drastically the wave function describing ^{12}C is changed by projection. The intrinsic ground state of this nucleus is generally considered to be strongly oblate and has a large admixture of undesired angular momenta ($\langle \bar{J}^2 \rangle \approx 6$) in both the Nilsson and HF models. In the laboratory system, however, the ground state of this nucleus like all other even-even nuclei appears as a strictly spherical 0^+ state. Nevertheless, the essential long range correlations remain after projection, which has been verified in the case of ^{12}C by comparison with shell-model results.

In the shell-model picture of light nuclei the isospin quantum number is generally assumed to be well-defined. This is also the case in the Nilsson model if, as in paper I, the potential parameters are chosen equal for both kinds of nucleons. Paper I uses the approximation that the neutrons are separately coupled to angular momentum zero and because of this the isospin invariance of the model state is lost. In order to restore this symmetry, and thus make a comparison with shell-model results possible, it is necessary to project isospin as well. This was done with the same PY-operator as for angular momentum.

4. The single-nucleon formfactor

The (p,2p) or (e,e'p) reactions, studied under experimental conditions where the impulse approximation is applicable, provide a sensitive instrument for nuclear structure investigations [8]. In the impulse approximation the cross section is essentially formed as a product of a free proton-proton (or electron-proton) cross-section times a nuclear structure factor. The latter is based on the single-nucleon form factor [9] calculated as the overlap integral $\langle \bar{r}, \phi(J_f, M_f, A-1) | \phi(J_i, M_i, A) \rangle$ between the initial A-particle state $\phi(J_i, M_i)$ and the final (A-1)-particle state $\phi(J_f, M_f)$ at the collision point $\bar{r}$. This quantity is the probability amplitude for finding the nucleon considered at the position $\bar{r}$ relative to the final nucleus in its state $\phi(J_f, M_f)$ when the whole system is in the initial state $\phi(J_i, M_i)$. It is therefore consistent to think of this overlap integral as the wave function in the target nucleus of the nucleon relative to the core in a specified state. It is worth observing that this interpretation holds irrespective of any model assumptions concerning the structure of the states $\phi(J_i, M_i)$ and $\phi(J_f, M_f)$. Consequently, it is a quantity that can be determined experimentally within the limitations set by the direct interaction picture of the reaction mechanism, and this is as close as one can come to the concept of a single-nucleon wave function as it is used in e.g. shell-model or Hartree-Fock theories. Paper I, which constitutes a large part of the present dissertation, deals with various aspects of this fundamental quantity, the single nucleon form factor.

With modern experimental techniques it is possible in a (p,2p)-experiment to resolve single-particle states with an uncertainty (ΔE) of less than one MeV. Together with the concept of collective rotation this provides the basic ingredient of a simple semi-classical argument for the necessity of using rotationally invariant states. As stated in paper I the reaction time ($\sim \Delta E/\hbar$) is long enough for the core to rotate as much as one radian during the reaction process. This may at first thought seem inconsistent with the direct reaction picture on which most theoretical analyses are based.

However, the direct interaction need not be the rapid process suggested e.g. by the term "impulse approximation". In a direct reaction only one, or at most a few, of the degrees of freedom of the initial state are changed in order to produce the final state. Clearly this requires that the strong interaction causing the change is selective with respect to these particular degrees of freedom rather than rapid. Therefore in the case of fast incident nucleons it is the short range of their interaction with the target nucleons, and the small de Broglie wavelength of the projectiles, which constitute the justification for the direct interaction picture. The energy resolution of the experiment, taken as a whole and thus including the energy spread of the beam as well as the energy acceptance widths of the detectors, requires that the reaction can be considered as a quasistationary process. In this picture the incident projectile is represented by a wave packet with a spatial extension $\sim \hbar v_{\text{inc}}/\Delta E_{\text{inc}}$ and the outgoing wave packets have extensions correspondingly related to the

detector acceptance widths ΔE_1 and ΔE_2 .

Starting from the empirically well-established Nilsson model and realizing the necessity of angular momentum projection to bring this model into accord with the symmetry requirements of the experimental situation, the PY-projection formalism was adopted for the calculation of single nucleon overlap integrals between intrinsically deformed states. Sometimes there are several low-lying intrinsic states from which it is possible to project the desired angular momentum and hence several possible candidates exist. In paper I this problem was encountered especially in the case of ^{11}B . In order to overcome this difficulty a phenomenologically acceptable residual force was diagonalized in the non-orthogonal basis consisting of the three lowest projected states. In this way the available empirical information, embodied in the phenomenologically determined parameters of the nuclear model and the residual interaction, was utilized within the limits set by general symmetry requirements. A fully satisfactory agreement with the experimental results was achieved for the reactions $\text{C}^{12}(\text{p}, 2\text{p})^{11}\text{B}$ and $^{16}\text{O}(\text{p}, 2\text{p})^{15}\text{N}$ as regards both the magnitude and the angular distributions of the cross-sections. In particular, the spectroscopic factors were found to depend significantly on the deformation of the intrinsic states. In addition, some projected states with significant statistical weight have no counterpart in the spherical shell model. Thus the inclusion of these long-range correlations is of utmost importance but as discussed above, to do this without restoring the rotational symmetry is beset with spurious consequences.

It has long been realized that the cross section is extremely sensitive to the behaviour of the radial distribution at large distances. In particular it has been found important to reproduce the exponential decrease governed by the separation energy ($E_i^A - E_f^{A-1}$) of the knocked-out proton. In paper I a special study has revealed that for form factors with the appropriate asymptotic behaviour there is a zone of strong sensitivity situated at about 5 fm, i.e. at about twice the value of the root mean square radius of the radial distribution in the case studied. In this region the deformation, by admixing components from higher shells, has quite a strong influence on the form factor, and hence also on the magnitude of the cross section.

There is no doubt that the data obtained in knock-out reactions of the type (p,2p) or (e,e'p) contain much information on especially the single-nucleon aspects of nuclear structure. In order to extract this information the analysis has so far been based on the Distorted Wave Impulse Approximation (DWIA). At intermediate energies, this model of the reaction mechanism seems to be adequate for the analysis of the experimental data obtained so far.

It is worth stressing again that the analysis here is made in terms of the form factor, which is conceptually closer to the recoil momentum distributions actually measured than to the single-nucleon wave functions furnished by the shell-model or Hartree-Fock theories. In this thesis the ambition has been to bridge the gap between these theories of nuclear structure and the measureable form factor. The remaining gap between the form factor and

the measured cross sections which is supposed to be covered by the DWIA with its possible shortcomings, is not considered here. This must, however, be kept in mind when the results of the calculations are confronted with the experimental data.

5. The cranking model

The cranking model, originally formulated 25 years ago by Inglis [10], is today widely used for describing rotational states in nuclei. In paper II, IV and V various aspects of the cranking model are scrutinized. A strong incentive for the formulation of the cranking model was the concept of an intrinsically deformed single-particle potential to be considered in a rotating frame of reference. Solutions of the cranking Hamiltonian $H' = H - \omega J_x$ describe the independently moving nucleons in a deformed single-particle potential rotating around the x-axis with the constant angular frequency ω [11]. The desired angular momentum J is accounted for in a schematic way by adjusting ω so as to fulfill the equality $\langle J_x \rangle = \sqrt{J(J+1)}$. In a self-consistent variational scheme the energy is minimized subject to the above constraint on J_x and hence ω takes the role of a Lagrangian multiplier.

In the HF-solution, say for an even-even heavy deformed nucleus, the total wave function contains a large number of significant components with high angular momentum. Although $\langle J_z \rangle = 0$, it is not the $J=0$ component of the wave function but rather one near $J=10$ that is the largest one.

This may result in a $\langle J^2 \rangle$ value exceeding one hundred units of $\hbar$. Consequently, high-spin states can be described by a non-rotating HF procedure combined with angular momentum projection. However, such calculations are computationally demanding and not necessarily the most accurate ones in describing rotational parameters such as the moment of inertia. In view of this it is not surprising that the cranking model has become so widely used. One can regard the cranking procedure as an economical way of excluding unimportant regions of the configuration space. It has, in fact, been characterized as an approximate projection method [12]. The wave function will generally be dominated by the component having its angular momentum J equal to $\langle J_z \rangle$, but still there is a relatively large spread in J .

A great step would be taken if the projection of angular momentum could be performed in a consistent way together with cranking calculations. This is of course only feasible for a small number of particles in a rather limited configuration space. Such restricted calculations are still interesting since there exists high-spin states also among the light nuclei. As the cranking wave function does not have a sharply defined K value the projection procedure is to some extent ambiguous. In addition it is more involved than when K is well-defined. In paper II a method is proposed for projection of angular momentum from cranking model wave functions. The method can easily be applied to rotational states in light nuclei for which comparison with the results of other models, e.g. the shell-model, can be performed.

Although the original Hamiltonian, H , may have a variety of spatial symmetries only two of them remain in the cranking Hamiltonian $H' = H - \omega J_x$, namely (i) parity and (ii) invariance with respect to a rotation $R_x(\pi)$ through an angle ω around the x-axis, provided, of course, that these symmetries are present in H . The latter symmetry gives rise to the signature quantum number α . The quantity α is related to the eigenvalue r of the rotation $R_x(\pi)$ by the relation $r = (-1)^\alpha$. In order to take advantage of these remaining symmetries calculations are usually performed in the "signature basis", obeying these invariance principles. For single-particle wave functions α is either $+1/2$ or $-1/2$, and for a many-particle configuration the total signature α_t is the sum of the single-particle values.

In the experimental analysis of rotational bands it has been found that the configurations assigned to the bands can generally be classified with respect to α_t according to the relation $J = \alpha_t \bmod 2$, where J denotes the spin values of the rotational band members [13,14]. This circumstance is a strong indication that α_t is a physically important quantum number. The same symmetry relation can also be found in the rotor model, a fact which may inveigle us into the false conception that the relation $J = \alpha_t \bmod 2$ also holds for the cranking solution. This is not the case as can be seen from the discussion in paper II. It is, however, an excellent approximation for the low-lying configurations adjacent to the yrast line in well-deformed nuclei. For more special excitations this relation may fail completely. In paper II the rotational axis, i.e. the x-axis, is chosen as the quantization axis

of angular momentum. It is then easily proven that the cranking solution obeys the symmetry relation $M_x = \alpha_{t \bmod 2}$ where M_x is the projection of angular momentum on the rotational axis. Hence it can be concluded that the constraint $\langle J_x \rangle = \sqrt{J(J+1)}$ can only be meaningful when the relation $J = \alpha_{t \bmod 2}$ is a good approximation.

6. Projection of particle number

Pairing correlations can be brought into the cranking model by employing the so called cranked Hartree-Fock-Bogoliubov (CHFB) approach [15,13]. In papers IV and V the CHFB method is used within the $i_{13/2}$ -shell in order to investigate various features of the CHFB-model. In both papers the particle number is projected, and as for the angular momentum projection an algebraic and exact technique has been developed. This facilitates the analysis of the results though again the applicability is confined to calculations in a small valence space. The number-projected wave functions are represented as uncoupled Slater determinants (the m-scheme) which makes a subsequent angular momentum projection possible.

Because of the cranking term the time reversal symmetry is lost, and hence the cranking wave function will not explicitly exhibit the convenient BCS-form [16]. Due to Bloch and Messiah [17] it is known, however, that there always exists a transformation which formally brings the CHFB solution in the BCS-form. Of course, in this case the u - and v -factors do not correspond to time-reversed states.

However, the formal advantages of the representation remain.

7. Quasiparticle levels in rotating nuclei

A small interaction at the point of closest approach between two rotational bands is today considered as the most probable mechanism behind an anomaly of the moment of inertia, usually referred to as the backbending effect (bb). This phenomenon is often found in rare earth nuclei [18]. The two bands, the ground (g-) and Stockholm (S-) bands, have different character in the sense that the S-band, unlike the g-band, has a large alignment of angular momentum. The backbending phenomenon was predicted in 1960 [19] more than ten years before it was experimentally detected [20]. Since then the interest in high spin studies has been mounting experimentally as well as theoretically.

The cranked Hartree-Fock-Bogoliubov (CHFB) [13] approach is the foremost and richest theoretical method for investigations of this and related phenomena. In the CHFB picture the bb can be understood as being due to the weak interaction between the zero quasiparticle (0qp) configuration and the first excited two quasiparticle (2qp) configuration. These configurations carry very different amounts of angular momentum and are strongly mixed in the crossing region, which introduces difficulties and spurious effects. Because of this the reliability of the model in the interaction region has been questioned [21]. However, it is only when one of the interacting quasiparticle levels

is unoccupied and the other occupied that problems arise. A crossing below the Fermi surface causes no problems, i.e. it introduces no "intruder" state.

In papers IV and V various aspects of quasiparticle levels in a rotating frame of reference are elucidated. An extended version of the cranked $i_{13/2}$ HFB model [21] is used, and this model is found to be enough structured to display a wealth of physically interesting features. The effects found in this restricted investigation will also be encountered in a full scale calculation although somewhat modified. The core levels (those not belonging to the $i_{13/2}$ shell) will gradually build up a collective angular momentum as ω increases and thus change the total wave function.

In paper IV a method is presented for elimination of the interaction between the 0qp and 2qp configurations applicable also for self-consistent calculations. The interaction is eliminated not only at the first crossing but also at the two or three following ones. It is then possible to follow a given configuration up to very high rotational frequencies without disturbances from other adjacent levels. The wave functions change only little when the rotational frequency increases from zero to very high values ($\sim$ three times the frequency where the first bb usually occurs). The self-consistent pairing gap (Δ) of the ground-band configuration remains large all the time and only a very small decrease with increasing frequency is found. Particle number projection makes the self-consistent Δ -values significantly larger especially for the lower 2qp configuration. The explanation is to be

sought in the increasing admixture of undesired particle numbers as Δ increases.

From the results in paper IV pairing correlations may be expected to have importance also for relatively high rotational frequencies. This is the case not only for the ground configuration but also for low-lying 2qp configurations. The pairing collapse of the yrast band, which is supposed to appear around spin $25\frac{1}{2}$, can be explained as being due to a 4qp configuration (relative to the ground configuration) becoming the lowest one. Paper IV further demonstrates the possibility of finding characteristic labels for each qp configuration in the form of certain Slater determinants in the projected wave functions. The growing interest for classifying experimental data in terms of quasiparticle configurations motivates an analysis of the kind presented in paper IV.

The oscillating character of the interaction strength between the 0qp and 2qp configurations as a function of the chemical potential is the issue in paper V. This feature has been much discussed since it was discovered in 1978 [22] which is natural in view of the far-reaching physical implications it may have. In the CHFB model the sharpness of backbending is directly related to the interaction strength. Therefore it would be very exciting if one could see a corresponding behaviour in the degree of backbending as a function of nucleon number for a series of isotopes or isotones. There are in fact indications of such a behaviour [22], and this has strengthened the confidence in model studies of this kind. Consequently, it is of great interest to get this behaviour of the

interaction strength further elucidated.

In order to shed light on the mechanism behind the oscillating behaviour of the interaction an investigation was undertaken starting from the $i_{13/2}$ CHFB model. In paper V it is shown how the changes of sign of the interaction matrix element between the two lowest quasi-particle configurations can be explained from certain phase relations between the different particle number components. It was found important to define the non-interacting levels in a consistent way. If this is done the single-particle term of the Hamiltonian has the largest matrix elements between the non-interacting configurations while the rotation term (ωj_x) presents very small values. Furthermore, if a definite particle number is projected out from both configurations it is possible to calculate the corresponding matrix element of the number conserving Hamiltonian. This quantity displays an oscillating character as a function of the chemical potential. It would be very interesting to see if these oscillations also are present in a full scale calculation, i.e. in a situation where particle number projection definitely would improve the wave functions.

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